

Initial (-3,1) Terminal (4,5)

$\langle 4 - (-3), 5 - 1 \rangle$

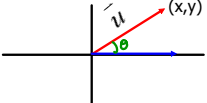
$\langle 7, 4 \rangle$

$7i + 4j$

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Direction Angle of a Vector

Let unit vector u have a terminal point (x,y) on the unit circle:



θ is the angle measured from the positive x-axis (initial side) to the vector (terminal side).

$x = \cos \theta$
 $y = \sin \theta$

$u = \langle \cos \theta, \sin \theta \rangle$
 $\cos \theta i + \sin \theta j$

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Suppose that u is a unit vector with direction angle θ .
 If $v = ai + bj$ is any vector that makes an angle θ with the positive x-axis, then it has the same direction as u and you can write:

$v = \text{length of } v \cdot \text{unit vector } u$
 $= \|v\| \langle \cos \theta, \sin \theta \rangle$
 $= \|v\| (\cos \theta i + \sin \theta j)$

Direction θ
 $\theta = \tan^{-1} \left(\frac{y}{x} \right) + A$

| | |
|--------------|--------------|
| $+180^\circ$ | $+0$ |
| $+180^\circ$ | $+360^\circ$ |

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Example

Find the magnitude and direction angle, θ , for $v = -12i + 15j$.

$\|v\| = \sqrt{(-12)^2 + (15)^2}$
 $= \sqrt{144 + 225}$
 $= \sqrt{369}$

Direction angle
 $\theta = \tan^{-1} \left(\frac{15}{-12} \right) + 180$
 $= 128.66^\circ$

p. 434, #63 (HW)

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Example
Find the magnitude and direction angle given

$$\vec{v} = 8(\cos 135^\circ \vec{i} + \sin 135^\circ \vec{j})$$

easy!

$$\|\vec{v}\| = 8$$

$$\theta = 135^\circ$$

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Example
Find the component form of the sum of \vec{u} and \vec{v} with direction angles $\theta_u = 30^\circ$ and $\theta_v = 90^\circ$ given $\|\vec{u}\| = 2$ and $\|\vec{v}\| = 2$

Note: \vec{u} is NOT a unit vector in this problem. The magnitude $\neq 1$.

$$\vec{u} = 2(\cos 30^\circ \vec{i} + \sin 30^\circ \vec{j})$$

$$= 2\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$$

$$= 2\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

$$\vec{u} = \langle \sqrt{3}, 1 \rangle$$

$$\vec{v} = 2(\cos 90^\circ \vec{i} + \sin 90^\circ \vec{j})$$

$$= 2(0\vec{i} + 1\vec{j})$$

$$= 2\langle 0, 1 \rangle$$

$$\vec{v} = \langle 0, 2 \rangle$$

p. 434, #73 (HW)

$$\langle \sqrt{3}, 1 \rangle + \langle 0, 2 \rangle$$

$$\langle \sqrt{3}, 3 \rangle \text{ or } \sqrt{3}\vec{i} + 3\vec{j}$$

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Let's talk through #81 together... Force = magnitude

| | | |
|----------------|----------------|------------------|
| <u>Force 1</u> | <u>Force 2</u> | <u>Resultant</u> |
| 45 lbs | 60 lbs | 90 |

Law of cosines

$$\chi = \cos^{-1}\left(\frac{45^2 + 60^2 - 90^2}{2(45)(60)}\right)$$

$$\chi = 117.3^\circ$$

$$180 - 117.3^\circ = 62.7^\circ$$

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HOMEWORK

...all of 6.3 will be due Monday; the underlined portion is what we went over today...

6.3 (p. 433): 1-33 (odd), 35-59 (odd) 61-81 (odd, omit 77)

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