7.6: Matrix Inverses

$$\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Real Numbers:

$$a \times \frac{1}{a} = 1$$

Matrices:

$$A \cdot A^{-1} = I$$

$$A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

sign sign

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a+4c=1$ $b+4d=0$

$-a-3c=0$ $-b-3d=1$

$a=-4c+1$

$b=-4d$

$$\begin{bmatrix} a+4c & b+4d \\ -a-3c & -b-3d \end{bmatrix}$$

$-(-4c+1)-3c=0$

$4c-1-3c=0$

$c=1$

$-(-4d)-3d=1$

$4d-3d=1$

$d=1$

Shortcut to Find the Inverse of a 2x2

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \cdot \frac{1}{ad-bc}$

2 Conditions:

1. Square matrix
2. $ad-bc \neq 0$
3. Calculator

$$\begin{array}{c c} A & B \\ \begin{bmatrix} x-2y+3z \\ -x+3y \\ 2x-5y+5z \end{bmatrix} & \begin{bmatrix} =9 \\ =-4 \\ =17 \end{bmatrix} \end{array}$$

$$A^{-1}\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}$$

$$A^{-1}AX = A^{-1}B$$

calculator

$$\begin{matrix} x \\ y \\ z \end{matrix}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$(1, -1, 2)$$

HOMEWORK

...you're learning more about matrices than Keanu Reeves himself...

7.6 (p. 547): 13,17,41,45, *49-59 odd

*write out the matrix equation $A^{-1}AX=A^{-1}B$

*if this method doesn't work, simply write "different method needed to solve" Do it

RREF

Notes 7.6 - Inverses

Identity Matrix: the nxn *square* matrix with 1's on its main diagonal and 0's everywhere else. (A square matrix has same number of rows & columns.)

EX: 4x4 identity matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

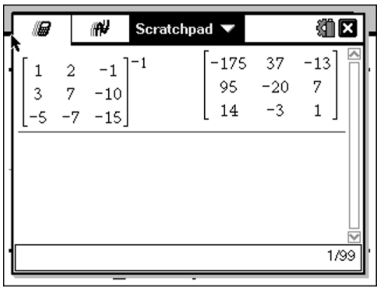
Inverse of a Square Matrix: the inverse matrix A^{-1} exists when

$AA^{-1}=I$ or $A^{-1}A=I$

- Only square nxn matrices can have inverses (called invertible or nonsingular).
- Non-square mxn matrices cannot have inverses and are called singular.
- Some square matrices do not have inverses. Your calculator will state "Error: Singular Matrix"

EX: Find the inverse A^{-1} using your calculator.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$$

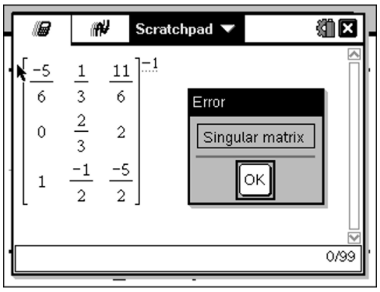


Simply raise the matrix to the -1 power.

EX: Find the inverse A^{-1} using your calculator.

$$A=\begin{bmatrix} -\frac{5}{6} & \frac{1}{3} & \frac{11}{6} \\ 0 & \frac{2}{3} & 2 \\ 1 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$$

The inverse does not exist!



Solving Systems with Inverses

We've learned to solve linear systems with the command *rref*.
Another way to solve the system of linear equations is to use inverses.

AX=B where A is a coefficient matrix, X is a solution matrix and B is a constant matrix.

system $\begin{cases} x-2y+3z=9 \\ -x+3y=-4 \\ 2x-5y+5z=17 \end{cases}$

$$\begin{array}{cccc} \text{coefficient matrix} & \text{solution matrix} & & \text{constant matrix} \\ \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} & = & \begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix} \end{array}$$

Proof
AX=B
$A^{-1}AX=A^{-1}B$ multiply inverse on both sides.
$IX = A^{-1}B$ yields identity matrix on left
$A^{-1}B = X$ flip equation

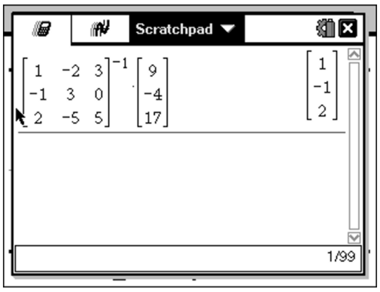
Rewrite the equation as $A^{-1}(B)=X$ to solve for (x,y,z).

$$\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rewrite the equation as $X = A^{-1}(B)$ to solve for (x,y,z).

$$\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Plug into calculator.



The solution matrix result tells us x=1, y=-1 and z=2. The solution to the system is (1,-1,2)

Show $A^{-1}AX=A^{-1}B$
Must do this on your test.

$$\begin{array}{ccccccccc} \text{inverse } (A^{-1}) & & \text{coefficients } (A) & & \text{variables } (X) & & \text{inverse } (A^{-1}) & & \text{constants } (B) \\ \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} & \cdot & \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} & \cdot & \begin{bmatrix} \quad \end{bmatrix} & = & \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} & \cdot & \begin{bmatrix} \quad \end{bmatrix} \end{array}$$

$$\begin{bmatrix} \frac{15}{2} & \frac{-5}{2} & \frac{-9}{2} \\ \frac{5}{2} & \frac{-1}{2} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\cdot\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} \frac{15}{2} & \frac{-5}{2} & \frac{-9}{2} \\ \frac{5}{2} & \frac{-1}{2} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\cdot\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}$$

Example - Page 589, Problem 52

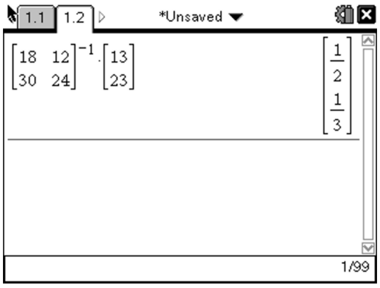
$$\begin{cases} 18x + 12y = 13 \\ 30x + 24y = 23 \end{cases} \qquad A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \qquad B = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

coefficients constants

Rewrite the equation as X = A^{-1}(B) to solve for (x,y,z).

$$X = \begin{bmatrix} 18 & 12 \\ 30 & 24 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 13 \\ 23 \end{bmatrix}$$

Order of multiplication is important!!



Therefore, x=1/2 and y=1/3 or the solution is (1/2, 1/3)

Show $A^{-1}AX=A^{-1}B$
Must do this on your test.

$$\begin{array}{ccccccccc} \text{inverse } (A^{-1}) & & \text{coefficients } (A) & & \text{variables } (X) & & \text{inverse } (A^{-1}) & & \text{constants } (B) \\ \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} & \cdot & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} & \cdot & \begin{bmatrix} \quad \\ \quad \end{bmatrix} & = & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} & \cdot & \begin{bmatrix} \quad \\ \quad \end{bmatrix} \end{array}$$

$$\begin{bmatrix} \frac{1}{3} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} 18 & 12 \\ 30 & 24 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} 13 \\ 23 \end{bmatrix}$$