Testing Block	Hour	Time
First Block	(1)	8:20am – 10:35
Second Block	(3)	10:40am – 12:50/**1:00**
12:50		Dismiss Juniors and Seniors
1:00		**Dismiss Sophomores**
1:35		ALL students report to third block
Third Block	(5)	1:40 – 3:50

BE READY TO DISCUSS

If I wanted to solve this system of linear equations, which of these operations would I be allowed to do? Which, if any, would be illegal ?

$$\begin{aligned} 2x - 3y &= 7 \\ x + 4y &= -2 \end{aligned}$$

*Multiply/divide both sides of an equation by any number
*Add/subtract the two rows
*Switch the order of the equations (put the bottom one on top, or vice versa)

STREAMLINING...

$$\begin{aligned} 2x - 3y &= 7 \\ x + 4y &= -2 \end{aligned}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & \text{ANS} \end{array} \\ \left[\begin{array}{cc|c} 2 & -3 & 7 \\ 1 & 4 & -2 \end{array}\right] \end{array} \quad 2 \times 3$$

A rectangular array of numbers like the one above is called a **matrix** (pl. **matrices**).

Dimension
The **order** of a matrix is number of rows x number of columns.
Thus, the matrix above has order 2x3.

System Operations	Row-Equivalent Operations
1. Interchange any two equations	1. Interchange any two rows
2. Multiply both sides of one of the equations by a nonzero constant	2. Multiply each entry in a row by the same nonzero constant
3. Add a nonzero multiple of one equation to another	3. Add a nonzero multiple of one row to another row

$$\begin{aligned} 2x - 3y &= 7 \\ x + 4y &= -2 \end{aligned}$$

For a system of equations in three variables, our goal is to find a row-equivalent matrix in the form

$x = 30$

$x + 2y + z = 10$

$-22 + 2$

$$x + ay + bz = c \qquad y + dz = e \qquad z = f$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{array}\right]. \end{array} \qquad \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 1 & 10 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \end{array}$$

$y = -11$

$y + 5z = -1$

$z = 2$

The variables can then be reinserted to form equations from which we can complete the solution.

Example 1

Solve the system of equations using Gaussian elimination.

$$x - 2y + 3z = -4$$
$$3x + y - z = 0$$
$$2x + 3y - 5z = 1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 3 & 1 & -1 & 0 \\ 2 & 3 & -5 & 1 \end{array}\right]$$

$z = 3$

$7y - 10(3) = 12$

$7y - 30 = 12$

$7y = 42$

$y = 6$

$x - 2y + 3z = -4$

$x - 2(6) + 3(3) = -4$

$x - 12 + 9 = -4$

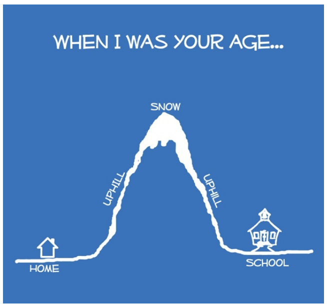
$x - 3 = -4$

$x = -1$

$(-1, 6, 3)$

GOAL

$$\left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{array}\right].$$



The procedure we did for Example 1 is called **Gaussian elimination with matrices**. The last matrix that we got is in **row-echelon form**.

An even better form for our last matrix would be something that looked like:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right].$$

The form of this matrix is called **reduced row-echelon form** and it is found by a method called **Gauss-Jordan elimination**.

Q: What solutions would this represent?
A: $x = a \quad y = b \quad z = c$

The unfortunate part is that it's hard enough to get a matrix in row-echelon form, as witnessed by Example 1. The good news is, we have calculators that can do it all for us.

Calculator Steps for Producing a Reduced Row-Echelon Matrix (TI 84)

1. 2^{ND} MATRIX
2. Scroll over to MATH
3. Select B: rref(***This stands for reduced row-echelon form*
4. ALPHA F3
5. Select order of matrix
6. Edit matrix
7. ENTER

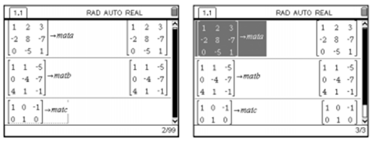
Entering a Matrix

Press [home] and choose Calculator. To access the $(m \times n)$ matrix template, press [ctrl] [matrix]. Highlight the small block, which pictures a 3×3 matrix, and press [click]. Enter the number of rows and columns and press [enter]. The handheld displays an empty matrix. Move to each element in the matrix and type the appropriate value in each cell.

To store the matrix as a variable, press ▶ until you exit the matrix, press [ctrl] [sto→ var], type the name of the matrix, and press [enter].

Editing a Matrix

To edit a matrix, highlight it and press [enter]. The matrix will appear in the entry line. Move to the elements you would like to edit and type new values. Then press [enter].



Example 2

Try Example 1 using your calculator.

$$x-2y+3z=-4$$
$$3x+y-z=0$$
$$2x+3y-5z=1$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 \\ 3 & 1 & -1 & 0 \\ 2 & 3 & -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

2nd, matrix, edit, [A], fill matrix
2nd, Quit, 2nd, matrix, math, [B] rref,
2nd Matrix [A]
Enter

Example 3

$$m+n+t=9$$
$$m-n-t=-15$$
$$3m+n+t=2$$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 1 & -1 & -1 & -15 \\ 3 & 1 & 1 & 2 \end{bmatrix}$$

No Sol.

$p+q+r=1$

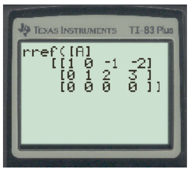
$p+2q+3r=4$

$4p+5q+6r=7$

infinite

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

$(z-2, -2z+3, z)$



$x-z=-2$

$\boxed{x=z-2}$

$y+2z=3$

$\boxed{y=-2z+3}$

HOMEWORK

...poor, poor Gauss...he had no graphing calculator and Mrs. Bybee

7.4 (pg. 521): 1-6 all, 7-13 odd, 49-59 odd, 61, 63

49-63 odd

Jackie