Questions on Quiz?

Jan 20-3:31 PM

Fundamental Theorem of Algebra

Theorem. Every polynomial function of degree n, with $n \geq 1$ has at least one zero in the set of complex numbers.

A result from the Fundamental Theorem of Algebra is this beauty...

Every polynomial function f of degree n, with $n \geq 1$ can be factored into n linear factors (not necessarily unique); that is,

$$f(x) = a_n(x - c_1)(x - c_2)\ldots(x - c_n).$$

Note that the c's may be real or complex and are not necessarily different from each other.

The proof of this theorem is beyond the scope of this course and actually is not algebraic in nature whatsoever...which makes the title of this theorem somewhat ironic.

So, what does the Fundamental Theorem tell us exactly...?

You have the same # of zeros as your degree. Some may be real and some may be non-real.

One helpful hint about complex zeros:

Come in pairs

Nonreal Zeros: a+bi and a-bi, b≠0

If a complex number $a+bi$, $b\neq 0$, is a zero of a polynomial function with **real** coefficients, then its complex conjugate, $a-bi$, is also a zero.

Example

Given that the following numbers are zeros of some function with real coefficients, what other numbers do we know are zeros?

[A] $-2+5i$

$-2-5i$

[B] $4i, -7i$

$-4i, 7i$

Example | p. 144, #16

Find all zeros of the function and write the polynomial as a product of linear factors. Use a graphing utility to graph the function to verify your results graphically.

$$f(x) = 81x^4 - 625$$

$$(9x^2 - 25)(9x^2 + 25)$$

$$(3x+5)(3x-5)(3x+5i)(3x-5i)$$

$$9x^2 = -25$$

$$\sqrt{x^2} = \sqrt{\frac{-25}{9}}$$

$$x = \pm \frac{5i}{3}$$

$$3x = \pm 5i$$

$$x = \frac{-5}{3}, \frac{5}{3}, \frac{-5i}{3}, \frac{5i}{3}$$

What if we couldn't factor...?

Jan 20-3:36 PM

Factoring//How far do I go?

Example

Factor: $x^4 + 2x^2 - 63$

- Irreducible over the rationals: $(x^2+9)(x^2-7)$
- Irreducible over the reals: $(x^2+9)(x+\sqrt{7})(x-\sqrt{7})$
- Completely factored: $(x+3i)(x-3i)(x+\sqrt{7})(x-\sqrt{7})$

p. 144, #47 (HW)

Example | p. 145, #52

The given function has 3i as a zero. How many more zeros are there? 2

How could you find those zeros?

$$f(x) = (x^3 + x^2) + (9x + 9)$$

$$x^2(x+1) + 9(x+1)$$

$$(x+1)(x^2+9)$$

$$(x+1)(x+3i)(x-3i)$$

$$-1,\ -3i,\ 3i$$

p. 145, #51 (HW)

Jan 20-3:46 PM

Example | p. 145, #40

Write a polynomial with least degree given the following zeros:

$-1, -1, 2+5i, 2-5i$

$(x - -1)$

-1 (multiplicity 2), 2+5i

$\boxed{(x+1)(x+1)}(x-(2+5i))(x-(2-5i))$

(x^2+2x+1) $(x-2-5i)(x-2+5i)$

$x^2-2x+\cancel{5xi}-2x+4-\cancel{10i}-\cancel{5xi}+\cancel{10i}+25$

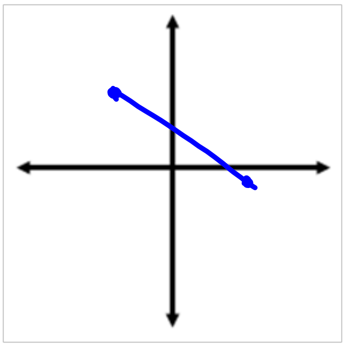
$(x^2+2x+1)(x^2-4x+29)$

$x^4-4x^3+29x^2+2x^3-8x^2+58x+x^2-4x+29$

$x^4-2x^3+22x^2+54x+29$

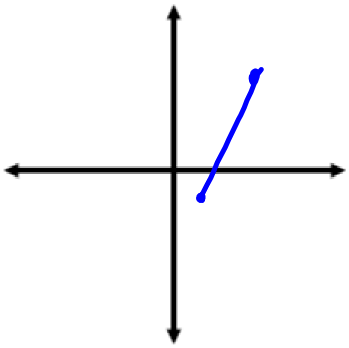
Intermediate Value Theorem

Sketch a graph of a continuous function *f* such that...



$f(-2)=3$ and $f(4)=-1$.

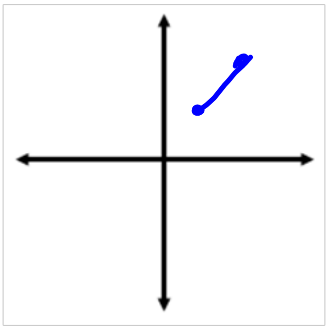
(-2,3) (4,-1)



$f(1)=-2$ and $f(3)=5$

Both of these functions crossed the x-axis.

Sketch a graph of a continuous function *f* such that f(2)=3, f(4)=5 that does NOT cross the x-axis.



Q: Why did the first two examples have to cross the x-axis, but the last one did not?

A: They must have opposite signs

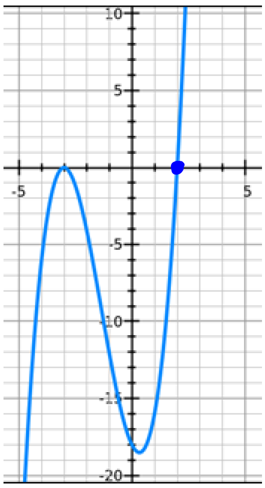
Jan 30-3:56 PM

Here's a consequence of the Intermediate Value Theorem that will come in handy...

For any polynomial function f, suppose that for $a \neq b$,

f(a) and f(b) are of opposite signs. Then the function has a real zero between a and b.

Consider the graph to the right.



f(1) is (positive/negative); f(3) is (positive/negative).
Therefore, there must be a real zero between 1 and 3.
Verify this statement by looking at the graph.

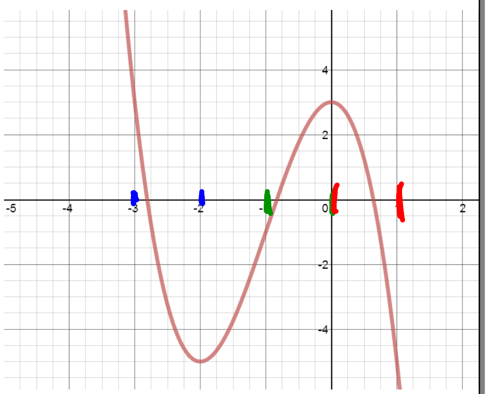
Example | p. 114, #80 (Use these directions for your HW.)
Use a graphing calculator to find any intervals of length 1 in which the polynomial function crosses the x-axis. Then use the Intermediate Value Theorem to prove the existence of these zeros.

$$f(x) = -2x^3 - 6x^2 + 3$$

$(-3, -2)$
$f(-3) = +$
$f(-2) = -$

$(-1, 0)$
$f(-1) = -$
$f(0) = +$

$(0, 1)$
$f(0) = +$
$f(1) = -$



Jan 20-3:58 PM

HOMEWORK

...existence of zeros

2.5

p. 144: 3, 11, 15, 23, 33, 39, 47, 51, 63;

p114 : 79, 81 (Use directions from class)

Jan 20-4:06 PM