Precalculus
WARM UP

Let $f(x)=x-30, g(x)=x+30$. *Find* $(f\circ g)(x)$ *and* $(g\circ f)(x)$.

$f(g(x))$
$f(x+30)=x+30-30=x$
$g(f(x))$
$g(x-30)=x-30+30=x$

Inverses

Jul 31-1:30 PM

56 $f(x)=\frac{1}{4}(x-1) \quad g(x)=4x+1$

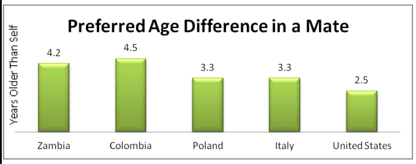
$f(g(x))$
$f(4x+1)=\frac{1}{4}(4x+1-1)=\frac{1}{4}(4x)=x$

$g(f(x))$
$g\left(\frac{1}{4}(x-1)\right)=4\left(\frac{1}{4}(x-1)\right)+1$
$=4\left(\frac{1}{4}x-\frac{1}{4}\right)+1$
$=x-1+1 \quad =x$

Jan 13-10:30 AM

Inverse of a Function

The following chart shows women's preferred age difference between themselves and their male mate in various countries.



Letting the countries be the domain, and the corresponding preferred years older than self be the range, write this data as a set of ordered pairs:

$f=\{(Z,4.2),(C,4.5),(P,3.3),(I,3.3),(U,2.5)\}$

Q: Is this a function? Why or why not?
A: yes

Now, reverse *f*, making the domain of *f* the new range and the range of *f* the new domain.

Q: Is this new relationship a function? Why or why not?
A: No

Feb 28-11:29 AM

If a function f is a set of ordered pairs (x, y), then we can "undo" f by reversing the components of all the ordered pairs. The result (y, x) may or may not be a function.

Example
Find the inverse of the relation for [A] and [B].

[A] $\{(1,2),(3,4),(5,6)\}$ [B] $\{(1,3),(2,3),(4,6),(5,6)\}$

Inverse
$\{(2,1),(4,3),(6,5)\}$
Function

Inverse
$\{(3,1)(3,2)(6,4),(6,5)\}$
Not Function

Feb 28-11:33 AM

Example

Consider the functions f and g below. [A] Describe what each one does to x in three steps. [B] Compute $g(f(2))$.

$f(x)=2x^3+1$

(1) cubed
(2) mult 2
(3) add 1

$g(x)=\sqrt[3]{\frac{x-1}{2}}$

(1) cube root
(2) Divide 2
(3) subtract 1

$g(f(2))=$

$f(2)=2(2)^3+1=17$

$g(17)=$ 2

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Definition. Let f and g be two functions such that

$f(g(\cancel{x}))=\cancel{x}$ 7 *for every x in g and*

$g(f(\cancel{x}))=\cancel{x}$ 13 *for every x in f.*

The function g is the inverse of the function f and is denoted by $\boxed{f^{-1}}$. So, $f(f^{-1}(x))=x$ & $f^{-1}(f(x))=x$.

The domain of f is equal to the range **of f^{-1} and vice versa.**

Note that $\boxed{f^{-1}(x)}$ does not mean $\cancel{\frac{1}{f(x)}}$ in general.

notation

Example

Show that f and g are inverses of each other. Notice that g "undoes" what f does to x.

$f(x)=4x-7, g(x)=\frac{x+7}{4}$ Find $f(g(x))$ and $g(f(x))$

$f(g(x))=4\left(\frac{x+7}{\cancel{4}}\right)-7$
$=x+7-7$
$=x$

$g(f(x))=\frac{(4x-\cancel{7})+\cancel{7}}{4}$
$=\frac{4x}{4}$
$=x$

Shows inverses

p. 69, #10 (HW)

Feb 28-11:39 AM

Finding the Inverse

(1) Replace f(x) with y
(2) Switch x and y
(3) Solve for y
(4) Replace y with $f^{-1}(x)$

Example

Find the inverse f^{-1} when:

[A] $f(x)=2x-7$

$y=2x-7$
$x=2y-7$
$+7 \quad +7$
$\frac{x+7}{2}=\frac{2y}{2}$
$\frac{x+7}{2}=y$

$f^{-1}(x)=\frac{x+7}{2}$

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[B] $f(x)=4x^{3/5}-1$

$y=4x^{\frac{3}{5}}-1$
$x=4y^{\frac{3}{5}}-1$
$x+1=4y^{\frac{3}{5}}$
$\left(\frac{x+1}{4}\right)^{\frac{5}{3}}=\left(y^{\frac{3}{5}}\right)^{\frac{5}{3}}$
$f^{-1}(x)=\left(\frac{x+1}{4}\right)^{\frac{5}{3}}$

[C] $f(x)=\frac{3}{x}$

$y=\frac{3}{x}$
$yx=\frac{3}{y}\cdot y$
$xy=3$
$y=\frac{3}{x}$
$f^{-1}(x)=\frac{3}{x}$

p. 70, #63 (HW)

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One-to-One Functions

Q: What happens if we try to find f^{-1} when $f(x)=x^2$?

A: $y=x^2$

$\pm\sqrt{x}=\sqrt{y^2}$

$\pm\sqrt{x}=y$

$f^{-1}(x)=\pm\sqrt{x}$

Notice that in f(x) we have the points

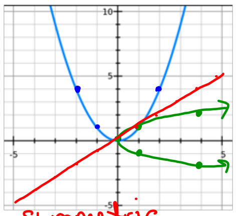
(-2, 4), (-1, 1), (1, 1), (2, 4). (0,0)

Flipping these points we get:

(4,-2)(1,-1)(1,1)(4,2)(0,0)

which does **not** represent a function.

(Why?) VLT



symmetric @ y=x

If f^{-1} is to be a function, then we can only have 1 **x-value for each y-value in** f.

Now y's can't repeat either!

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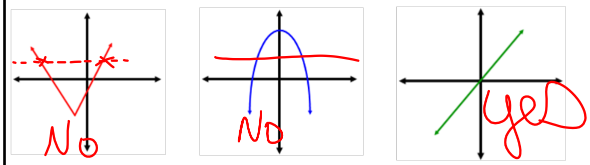
A function f has an inverse that is a function if there is no horizontal line that intersects the graph of the function f at more than one point. Which of the following have inverse functions?



Think about why the horizontal line test guarantees an inverse function. If f is a function then each x is unique for every (x, y). But f^{-1} reverses the x's and y's, so now each y must also be unique for every (x, y) in f.

Feb 28-11:48 AM

Definition. A one-to-one function is a function in which no two different ordered pairs have the same second component. Only one-to-one functions have unique y-values

Q: If (a, b) is in f, what point do we know is in f^{-1}?

A: (b, a)

The graph of f^{-1} is a reflection of the graph of f **about the line** y=x.

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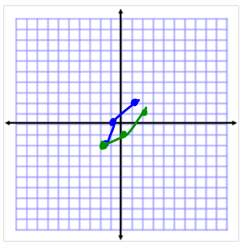
Example

The graph of a function f consists of two line segments: one from (-2,-2) to (-1, 0) and one from (-1, 0) to (1, 2). Graph f and f^{-1}.

(-2,-2)

(0,-1)

(2,1)



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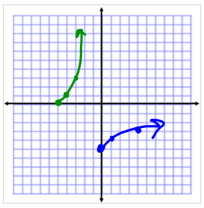
Example

Restrict the domain of the function f so that the function is one-to-one and has an inverse function. Find the inverse function of f, graph f and f^{-1}, and state the domain and range of f and f^{-1} when $f(x)=(x+5)^2$

$f(x)$	$f^{-1}(x)$
(-5,0)	(0,-5)
(-4,1)	(1,-4)
(-3,4)	(4,-3)



p. 70, #73 (HW)

$f(x)$

D: $[-5,\infty)$ D: $[0,\infty)$

R: $[0,\infty)$ R: $[-5,\infty)$

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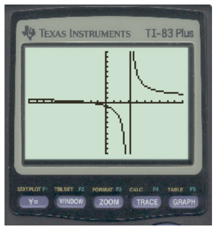
Example

Use the graph of to determine whether the function is one-to-one. If the function is not one-to-one, how could we restrict the domain so that is it one-to-one?

$[A]\ f(x)=\frac{x+2}{x-3}$



f(x) domain is $f^{-1}(x)$ range

f(x) range is $f^{-1}(x)$ domain

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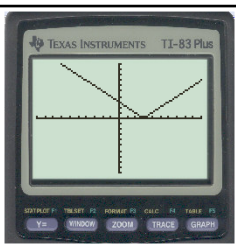
$[B]\ f(x)=|x-3|$



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Example

Find the inverse of $f(x)=\sqrt[4]{x-2}$ by thinking about the operations of the function and then reversing, or undoing, them.

Feb 28-11:58 AM

Q: What is the inverse of a function?
A:

l a s r e v e r

r e v e r s a l

Feb 28-12:00 PM

HOMEWORK

...inverse functions

1.6 (p 69): 9, 21-24, 29, 31, 59-67 (odd; just find the inverse function), 69, 77, 79, 81-87odd

Aug 1-10:32 AM