WARM UP

Consider the following expansions:

Note to self: kinda time-consuming

$(x+y)^0 = 1$

$(x+y)^1 = x^1 + y^1$

$(x+y)^2 = x^2 + 2x^1y^1 + y^2$

$(x+y)^3 = x^3 + 3x^2y^1 + 3x^1y^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + y^4$

$(x+y)^5 = x^5 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + y^5$

Discuss with someone around you:

What do you notice? What do you wonder? Be prepared to share.

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Before we get there... $_nC_r = \frac{n!}{(n-r)!r!}$

$_8C_2 = \frac{8!}{(8-2)!2!} = \frac{8!}{6!2!} = \frac{48 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdots}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 28$

$\binom{10}{3} = {}_{10}C_3 = 120$

p. 624, #1,3 (HW)

Note: 0!=1, by definition

Calculator commands:

TI-84: MATH-PRB-3

TI-Nspire: Menu-#5-#3

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Binomial Coefficients

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

${}_4C_0 \quad {}_4C_1 \quad {}_4C_2 \quad {}_4C_3 \quad {}_4C_4$

The coefficient of $x^r y^{n-r}$ is...

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Example

Expand the following binomials.

$(x+4)^3 = {}_3C_0x^3 \quad {}_3C_1x^24^1 \quad {}_3C_2x4^2 \quad {}_3C_34^3$

$1x^3 + 3x^2 \cdot 4 + 3x \cdot 16 + 1 \cdot 64$

$x^3 + 12x^2 + 48x + 64$

$(x-2y)^4 =$

$1x^4 \quad 4x^3(-2y)^1 \quad 6x^2(-2y)^2 \quad 4x^1(-2y)^3 \quad 1(-2y)^4$

p. 624, #17 (HW)

$x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$

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Example
Find the coefficient of the term a^6b^5 in the expansion of $(2a-5b)^{11}$.

$_{11}C_0$ $_{11}C_1$ $_{11}C_2$ $_{11}C_3$ $_{11}C_4$ $_{11}C_5(2a)^6(-5b)^5$

Next year: A few board races with these?

$462 \cdot 64 \cdot -3125$

$-92{,}400{,}000a^6b^5$

p. 624, #61 (HW)

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Pascal's Triangle (p. 623)

$_0C_0$

$_1C_0$ $_1C_1$

$_2C_0$ $_2C_1$ $_2C_2$

$_3C_0$ $_3C_1$ $_3C_2$ $_3C_3$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

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Example | p. 625, #70
Expand the binomial by using Pascal's Triangle to determine the coefficients.

$(5v-2z)^4$

$1(5v)^4 + 4(5v)^3(-2z) + 6(5v)^2(-2z)^2 + 4(5v)^1(-2z)^3 + 1(-2z)^4$

$625v^4 - 1000v^3z + 600v^2z^2 - 160vz^3 + 16z^4$

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Given: $f(x) = x^5$

Find: $f(x+h)$

$(x+h)^5$

$x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$

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HOMEWORK

...binomial theorem + pascal = <3

Due Monday |

8.5 (p624): 1-17 odd, 25,29,57-63 odd 69,71, 77,78,83,107

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NOTES CH 8.5 - BINOMIAL THEOREM

The Binomial Theorem

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

Example 1: Expand $(a+b)^4$.

Using the summation formula, it would look like the following:

$$(a+b)^4 = \sum_{r=0}^{4} {}_4C_r a^{4-r} b^r = {}_4C_0 a^4 b^0 + {}_4C_1 a^3 b^1 + {}_4C_2 a^2 b^2 + {}_4C_3 a^1 b^3 + {}_4C_4 a^0 b^4$$

Note: All the powers of "a" (my 1st term) start with n=4 and <u>decrease</u> to n=0.
All the powers of "b" (my 2nd term) start with n=0 and <u>increase</u> to n=4.
The sum of both powers always equals n=4.
The total number of terms in the expansion is n+1.

Calculate ${}_nC_r$ where ${}_nC_r = \frac{n!}{(n-r)!\,r!}$

${}_4C_0 =$ ${}_4C_1 =$ ${}_4C_2 =$ ${}_4C_3 =$ ${}_4C_4 =$

Instead of calculating ${}_nC_r$ for each term, sometimes it is easier to use Pascal's Triangle.
Note how row 4 of Pascal's Triangle matches the values above.

<u>Pascal's Triangle</u>
The top row is called "Row 0". Rows start and end with 1. Middle terms are found by adding two terms in the row above. Fill in Pascal's Triangle for rows 5, 6 and 7.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

____

____

____

Example 2: Expand $(x+1)^7$ using the same pattern as Ex. 1 and row 7 of Pascal's Triangle.

$(x+1)^7 =$

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Example 3: Expand $(x+2)^6$ using row 6 of Pascal's Triangle.

$(x+2)^6 =$

Example 4: Expand $(x-2y)^5$ using row 5 of Pascal's Triangle. Simplify.

$(x-2y)^5 =$

Example 5: Find a, the coefficient of ax^4 in the expansion of $(x+3)^7$.

The power of the binomial is 7, so n=7.
Any term in the expansion is defined as ${}_nC_r \cdot x^{n-r} 3^r$

Find the value of r by solving:

n − r = power of term given
7 − r = 4
so, r=3

Substitute n= 7 and r = 3 to find the coefficient of X^4

${}_7C_3 x^{7-3} 3^3$

Simplify.

$${}_7C_3 x^{7-3} 3^3 = 35x^4 \cdot 27$$
$$= 945x^4$$

Therefore the coefficient of x^4 is 945.

Exercise 6: Find a, the coefficient of ax^5y^4 in the expansion of $(x+2y)^9$

Plug into ${}_nC_r x^{n-r}(2y)^r$

n=______ r=_______

Coefficient of x^5y^4 = ________.

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