Questions over 8.3a?

31

$a_2 = -18 \quad a_5 = \frac{2}{3} \quad n = 6$

$a_5 = a_2(r)^3$

$\frac{1}{-18} \cdot \frac{2}{3} = -18 r^3 \cdot \frac{1}{-18}$

$\sqrt[3]{\frac{-1}{27}} = \sqrt[3]{r^3}$

$-\frac{1}{3} = r$

$a_2 = a_1 r$

$-3 \cdot -18 = a_1\left(-\frac{1}{3}\right) \cdot \frac{-3}{1}$

$54 = a_1$

$a_n = 54\left(-\frac{1}{3}\right)^{n-1}$

$a_6 = 54\left(-\frac{1}{3}\right)^{6-1}$

$a_6 = -\frac{2}{9}$

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8.3b: Geometric Series

Just as with arithmetic sequences, we would like to find a formula for the sum of the first n terms $a_1, a_2, a_3, \ldots, a_n$.

$S_n = a_1 +$

$S_n r = a_1 r +$

$S_n - S_n r =$

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Finite

The sum of the first n terms of a geometric sequence is given by:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

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Example

$\frac{40}{10} = 4 \quad \frac{160}{40} = 4$

Find the sum of the first 7 terms of the geometric series 10+40+160+...

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

$S_7 = \frac{10(1 - 4^7)}{1 - 4} = 54,610$

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Example

Find the indicated sum: $\sum_{k=1}^{3}\left(\frac{1}{2}\right)^{k}$.

$$\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$\frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$$

p. 607, #45 (HW)

Calculator Commands

Recall that you can use the sum and sequence commands on your calculator (under 2ND LIST)

```
sum(seq((1/2)^X,
X,1,3))▸Frac
               7/8
```

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Infinite Geometric Series

The sum of the terms of an infinite geometric sequence is an **infinite geometric series**. For some geometric sequences, S_n gets close to a specific number as n gets large. For example, consider the infinite series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \quad 1$$

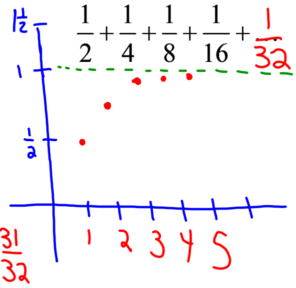
$S_1 = \frac{1}{2}$

$S_2 = \frac{3}{4}$

$S_3 = \frac{7}{8}$

$S_4 = \frac{15}{16}$

$S_5 = \frac{31}{32}$



click

GeoGebra

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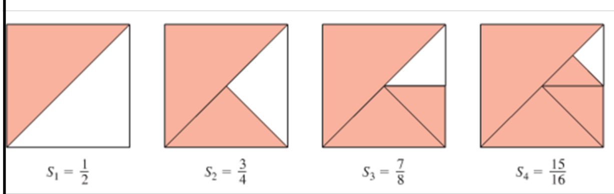
We say that 1 is the **limit** of S_n and also that 1 is the **sum of the infinite geometric sequence**, denoted by S_∞.



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Q: Will all infinite geometric sequences have a sum? If so, why? If not, give a counterexample.

A: NO → $1 + 5 + 25 + 125 + \ldots$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

Q: What must be true about the common ratio r for the sum of an infinite geometric sequence to exist?

A: $|r| < 1$

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Finding a formula for the sum of an infinite geometric series...

Finite $S_n = \frac{a_1(1-r^n)}{1-r}$.

Q: If $|r|<1$, what happens to r^n as n approaches infinity (aka as n gets very large)?
A: $S_\infty = \frac{a_1}{1-r}$ $|r|<1$

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When $|r|<1$, the limit or sum of an infinite geometric series is given by:

$$S_\infty = \frac{a_1}{1-r}$$

⇒Notice how this varies from S_n (i.e., the quantity $(1-r^n)$ is no longer in the numerator).

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Example
Find the sum, if it exists.

[A] $45+15+5+\ldots$ $\frac{15}{45}=\frac{1}{3}$ $\frac{5}{15}=\frac{1}{3}$ $|\frac{1}{3}|<1$ yes $S_\infty = \frac{45}{1-\frac{1}{3}} = \frac{135}{2} = 67.5$

[B] $\sum_{k=1}^{\infty} 5(0.4)^{k-1}$ $S_\infty = \frac{5}{1-0.4} = \frac{25}{3}$ $|0.4|<1$

[C] $\sum_{n=1}^{\infty} \pi^n$ No Sum $|\pi|<1$

p. 607, #59, 69 (HW)

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Do you remember compound interest?

Formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$

See page 606 Study Tip

Compounding Periods

Yearly: n=1
Semi-annually: n=2
Quarterly: n=4
Monthly: n=12
Weekly: n=52
Daily: n=365

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HOMEWORK

...geometric=multiply

Due Monday

8.3 (p. 607): 1-23 (1, 3, 5's); 27-35 (odd);
41, 45, 49, 55-58 (all) 59, 63, 69, 71, 77, 79

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[B] 6.131313… or $6.\dot{1}3$

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Applications

Example 7 From p. 642, Example 8
Say someone offered you a job for the month of September (30 days) under the following conditions. You will be paid 1 penny for the first day, 2 pennies for the second, 4 pennies for the third, and so on, doubling your previous day's salary each day. How much would you earn? Would you take the job?

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Example 8
Roberto borrows $15,000. The loan is to be repaid in 15 years, 10.5% interest, compounded annually. How much will be repaid at the end of 15 years?

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Example 9

A bungee cord rebounds 80% of the height jumped. This cord stretches to a length of 210 feet. After jumping and rebounding 9 times, how far has the bungee jumper traveled upward (the total rebound distance)? How far will the jumper have traveled upward before coming to a rest?

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