Questions over 8.1?

$3-9+27-81+243-729$

$$\sum_{n=1}^{6} 3^n(-1)^{n-1}$$

8.2: Arithmetic Sequences and Series

Review

Recall that a sequence that is defined **recursively** is a sequence where each term is dependent on the one(s) before it. Such as:

$$a_1 = 2;\ a_{n+1} = a_n + 3.$$

Example 1
List the first 5 terms of the above recursive sequence.

$a_1=2$ $a_2=5$ $a_3=8$ $a_4=11$ $a_5=14$

Arithmetic Sequences

The previous example is called an **arithmetic sequence**, which is a sequence such that each term after the first is found by adding the same number to the preceding term. The number that is added each time is called the **common difference** and is typically denoted by *d*.

Q: What was the common difference, *d*, for Example 1?
A: $d=3$

Example 2
For each of the following arithmetic sequences, identify the first term, a_1, and the common difference, *d*.

[A] $5,7,9,11,13,\ldots$ $a_1=5$ $d=2$

[B] $2,1,0,-1,-2,\ldots$ $a_1=2$ $d=-1$

[C] $1,\frac{17}{16},\frac{9}{8},\frac{19}{16},\frac{5}{4},\ldots$ $a_1=1$ $d=\frac{1}{16}$

$\frac{16}{16},\frac{17}{16},\frac{18}{16},\frac{19}{16},\frac{20}{16}$

p. 598, #1-7 (odd) (HW)

Thus, in general, arithmetic sequences with a common difference *d* take the form:

$a_{n+1}= a_n + d$

a_1

$a_2= a_1+d$

$a_3= a_1+2d$

$a_4= a_1+3d$

The *n*th term of an arithmetic sequence is given by $a_n= a_1+(n-1)d$ for any integer $n\geq 1$.

Example

Find a formula for a_n if a_1=15 and d=4.

$a_n=a_1+(n-1)d$

$a_n=15+(n-1)(4)$

$a_n=15+4n-4$

$\boxed{a_n=11+4n}$

p. 598, #21 (HW)

Example

Find a formula for a_n if a_1=-4 and a_5=16.

$a_5=a_1+4d$

$16=-4+4d$

$20=4d$

$5=d$

$a_n=-4+(n-1)(5)$

$a_n=-4+5n-5$

$\boxed{a_n=-9+5n}$

p. 598, #23 (HW)

Example

Find a formula for a_n if a_6=-38 and a_{11}=-73.

$a_{11}=a_6+5d$

$-73=-38+5d$

$-35=5d$

$-7=d$

$a_6=a_1+(6-1)(-7)$

$-38=a_1+(5)(-7)$

$-38=a_1-35$

$-3=a_1$

$a_n=-3+(n-1)(-7)$

$a_n=-3-7n+7$

$\boxed{a_n=4-7n}$

HOMEWORK

...arithmetic=add.

8.2a (p. 598): 1-33 (odd)

WARM UP

Recall the formula for finding the general term of an arithmetic sequence: a_n=____________.

Use this formula to work exercises 35-41 (odd); these are part of tonight's homework.

Arithmetic Series

Q: What's a series?

A: Sum of a Sequence.

Two ways to notate a series:

sigma notation $\sum$

S_n

$$1+2+\ldots+49+50+51+52+\ldots 99+100$$

101

101

101

101

$(50)101$

5050

p. 599, #57 (HW)

Example $S_n = \frac{n}{2}(a_1 + a_n)$ $a_n = a_1 + (n-1)d$

Find the sum of the first 65 terms of the arithmetic series

$$33 + 39 + 45 + 51 + \ldots$$

$S_{65} = \frac{65}{2}(33 + 417)$ $a_{65} = 33 + (65-1)(6)$

$= 417$

$\boxed{S_{65} = 14{,}625}$

p. 599, #63 (HW)

Example

Find the sum of the arithmetic series $\sum_{i=1}^{10}(6i-4)$

$S_{10} = \frac{10}{2}(2 + 56)$

$i = 1$ $6(1) - 4 = 2$

$i = 10$ $6(10) - 4 = 56$

$\boxed{S_{10} = 290}$

p. 599, #73 (HW)

Example

How many poles will be in a stack of telephone poles if there are 50 in the first layer, 49 in the second, and so on, with 6 in the top layer?

$S_{45} = \frac{45}{2}(50 + 6)$ $a_n = a_1 + (n-1)d$

$6 = 50 + (n-1)(-1)$

$S_{45} = 1260$ poles

$6 = 50 - n + 1$

$6 = -n + 51$

$-45 = -n$

$45 = n$

HOMEWORK

...arithmetic=add.

8.2b (p. 598): 35-41 (odd), 57-73 (odd), 81, 83