WARM UP

Each pair get a whiteboard, marker, and eraser.

We are going to see if you can read my mind and figure out a pattern's rule I am thinking about.
You will write down 3 numbers and I will tell you yes or no if your numbers follow my pattern, then you will discover my rule.

http://samjshah.com/2014/12/10/2-4-8/

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8.1: Sequences and Series
Say that a virus contaminates an initial population of 1,000 people. The virus spreads and on the second day 2,000 people are infected; the third day 4,000 are infected, and so on, as the number of people infected doubles each day. We can write this data as the following sequence of numbers:

1	2	3	4	5
↓	↓	↓	↓	↓
1,000	2,000	4,000	8,000	16,000...

We can think of sequences as functions that have the set of positive integers as their domain. A sequence can be either infinite or finite in length.

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Definition. A sequence is an ordered list of numbers. The first number (or "term") in the sequence is typically denoted by a_1, the second by a_2, and so on. In general then, the nth term is denoted by a_n.

Example 1
Find the first 4 terms (aka numbers) and the 20th term of the sequence given by $a_n = 5n - 3$.

$a_1 = 5(1) - 3 = 2$
$a_2 = 5(2) - 3 = 7$
$a_3 = 5(3) - 3 = 12$
$a_4 = 17$

$a_{20} = 5(20) - 3$
$= 97$

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Can you find the rule for the following:

1 2 3 4 5
1. $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25} \ldots$

1 2 3 4 5
2. $\frac{2}{1}, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9} \ldots$

$a_n = \frac{1}{n^2}$

$a_n = \frac{n+1}{2n-1}$

Explicit

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Recursive Definitions

A sequence may be defined **recursively** or by using a **recursion formula**. Such a definition lists the first term, or first few terms, and then describes how to determine the remaining terms form the given term(s).

Example 2

Fill in the following list for sequence:

$a_1, a_2, a_3, \ldots$ a_{n-2}, a_{n-1}, a_n, a_{n+1}, a_{n+2}, ...

Q: What do the dot, dot, dots mean? What's the term before a_n? After it?

A: Continue a_{n-1} a_{n+1}

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$$a_n = a_{n-1} + a_{n-2}$$

$a_3 =$

IN WORDS: The 3rd term equals the 2nd term plus the 1st term.

$a_4 =$

IN WORDS: The 4th term equals the 3rd term plus the 2nd term.

$a_9 =$

IN WORDS: The 9th term equals the 8th term plus the 7th term.

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$$\boxed{a_{n+1}} = a_n + a_{n-1}$$

a_{n-1} a_n a_{n+1}

$a_4 =$

IN WORDS: The 4th term equals the 3rd term plus the 2nd term.

$a_9 =$

IN WORDS: The 9th term equals the 8th term plus the 7th term.

If a_1=1 and a_2=1, then this sequence is called the "Fibonacci sequence."

1, 1, 2, 3, 5, 8, 13, ...

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Example

List the first 5 terms where a_1=25 and $a_{k+1} = a_k - 5$

a_1 a_2 a_3 a_4 a_5

25, 20, 15, 10, 5

p. 588, #57 (HW)

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Series: a sum of terms in a sequence

Sequence: 2, 4, 6, 8, 10

Series 2 + 4 + 6 + 8 + 10 (finite)

2 + 4 + 6 + 8 + 10 + ... (infinite)

A series can be condensed using sigma notation.

end

$$\sum_{n=1}^{3} n^2 - 1 =$$

Formula

1st #

$a_1 = 1^2 - 1 = 0$

$a_2 = 2^2 - 1 = 3$

$a_3 = 3^2 - 1 = 8$

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TI-Nspire: Use catalog, tab 4, find sigma button in middle of 2nd row.

TI-84: Alpha F2, #2.

TI-83: sum(seq(rule, index variable, lower limit, upper limit, 1))

You can find "sum" under 2nd List/Math and "seq" under 2nd List/OPS

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sum(seq(X^2-1,X,
1,3)
                11
```

*p. 585 for Properties of Sums

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Example

Use sigma notation to write the sum. Use a calculator to find the sum.

A) $\frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + ... + \frac{5}{1+15}$

$\sum_{n=1}^{15} \frac{5}{1+n} = 11.904$ or $\frac{108097}{9081}$

B) $0 + 7 + 26 + 63 + 124 + ... + 728$

$\sum_{i=1}^{9} i^3 - 1 = \underline{\underline{2016}}$

C) 1-2+3-4+5-6+7...+13

$(-1)^{x-1} x$

$\sum_{x=1}^{13} (-1)^{x-1} x = 7$

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HOMEWORK

...sequences and series

8.1 (p. 587):

1-17 (eoo), 21-25 odd, 33, 45, 47, 49, 57-63 odd, 71-79 odd, 83, 95, 99

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EX. Expand the series in sigma notation and find the sum.

A) $\sum_{i=1}^{3} \frac{3}{10^i}$

B) $\sum_{i=0}^{5} 3i^2$

C) $\sum_{i=3}^{7} (2i+1)$

terms in expansion = upper limit - lower limit +1

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Factorials

Examples

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$n! = (n)(n-1)(n-2)(n-3)....$

$(n+2)! = (n+2)(n+1)(n)(n-1)(n-2)....$

$0! = 1$

Simplify the factorial expression:

$\frac{(2n+2)!}{(2n)!}$

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