(33) $\vec{u} = \langle -12, 30 \rangle$ $\quad \frac{30}{-12} = -\frac{5}{2}$ $\quad \|\vec{u}\| = \sqrt{(-12)^2 + 30^2} = \sqrt{1044}$

$\vec{v} = \langle \frac{1}{2}, -\frac{5}{4} \rangle$ $\quad \frac{-\frac{5}{4}}{\frac{1}{2}} = -\frac{5}{2}$ $\quad \|\vec{v}\| = \sqrt{(\frac{1}{2})^2 + (-\frac{5}{4})^2} = \sqrt{\frac{29}{16}}$

$\vec{u} \cdot \vec{v} = \underset{-\frac{12}{2}}{-6} + \frac{-75}{2} = -\frac{87}{2}$

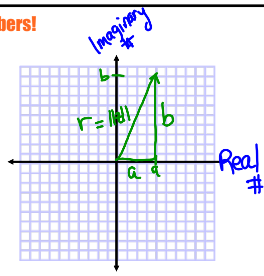
$\cos^{-1}\left(\frac{-\frac{87}{2}}{\sqrt{1044} \cdot \sqrt{\frac{29}{16}}}\right) = 180°$ Parallel

Nov 9-10:34 AM

The Complex Plane: Plotting Imaginary Numbers!

$z = a + bi$

real # imaginary #



Definition. The absolute value of a complex number z=a+bi is give by:

$r = \sqrt{a^2 + b^2}$

Distance from the origin to complex point

p. 456: #5 (HW)

Dec 4-2:29 PM

Trigonmetric Form of a Complex Number
(also called Polar Form)

$\cos\theta = \frac{a}{r}$

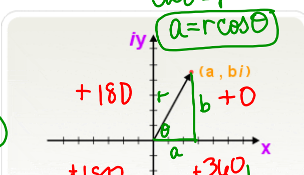
$\boxed{a = r\cos\theta}$

$z = a + bi$

$z = r\cos\theta + i r\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

$z = r \operatorname{cis}\theta$



$\sin\theta = \frac{b}{r}$

$\boxed{b = r\sin\theta}$

r is called the modulus of z;
θ is called the argument of z.

$r = \sqrt{a^2 + b^2}$ $\qquad \theta = \tan^{-1}\left(\frac{b}{a}\right) + A$

Dec 4-2:40 PM

Example 2 p. 449
Write the complex number in trigonometric form.

+180

$z = -2 - 2\sqrt{3}i$ $\qquad a = -2 \quad b = -2\sqrt{3}$

$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2}$

$= \sqrt{4 + 12}$

$= \sqrt{16}$

$= 4$

$\theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{-2}\right) + 180°$

$\theta = 240°$

$z = 4(\cos 240° + i\sin 240°)$

or

$z = 4 \operatorname{cis} 240°$

p. 456, #13 (HW)

Dec 4-2:47 PM

Example 3 p. 450

Write the complex number in standard form a+bi.

$$z=\sqrt{8}[\cos(-\frac{\pi}{3})+i\sin(-\frac{\pi}{3})]$$

$\sqrt{8}$, $2 \wedge 4$, $2\ 2$, $2\sqrt{2}$

$24 \wedge 2, 12 \wedge 2, 6 \wedge 2, 3$

$$z=\sqrt{8}\left(\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$z=\frac{\sqrt{8}}{2}+\frac{-\sqrt{24}}{2}i$$

$$\frac{\cancel{2}\sqrt{2}}{\cancel{2}}+-\frac{\cancel{2}\sqrt{6}}{\cancel{2}}i$$

Check on calculator

$$\boxed{z=\sqrt{2}-\sqrt{6}\,i}$$

p. 456, #37 (HW)

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Multiplying and Dividing Complex Numbers in Trig Form

$z_1=r_1(\cos\theta_1+i\sin\theta_1)$, $z_2=r_2(\cos\theta_2+i\sin\theta_2)$

$$z_1\cdot z_2=r_1\cdot r_2\left(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right)$$

$$\frac{z_1}{z_2}=\frac{r_1}{r_2}\left(\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right)$$

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Example 4 p. 451 $i^2=-1$

Find the product z_1z_2. Both forms have been given.

$\frac{2\pi}{3}+\frac{11\pi}{6}$

$\frac{4\pi}{6}+\frac{11\pi}{6}=\frac{15\pi}{6}$

In Standard Form:

$z_1=-1+\sqrt{3}i$

$z_2=4\sqrt{3}-4i$

In Trig Form:

$z_1=2(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$

$z_2=8(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6})$

$$z_1z_2=(-1+\sqrt{3}i)(4\sqrt{3}-4i)$$

$$-4\sqrt{3}+4i+12i+4\sqrt{3}$$

$$\boxed{16i}$$

$$16\left(\cos\left(\frac{2\pi}{3}+\frac{11\pi}{6}\right)+i\sin\left(\frac{2\pi}{3}+\frac{11\pi}{6}\right)\right)$$

$$16\left(\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}\right)$$

$$16(0+1i)$$

$$\boxed{16i}$$

p. 457, #55 (HW)

Dec 4-3:03 PM

Example (p. 457, #73)

$$\frac{0-2i}{\sqrt{3}-i}$$

$\boxed{z_1=2\,cis\,270^\circ}$

$\boxed{z_2=2\,cis\,330^\circ}$

$r_1=\sqrt{0^2+(-2)^2}=2$

$\theta_1=\tan^{-1}\left(\frac{-2}{0}\right)=270^\circ$

In Standard Form:

$$\frac{-2i}{(\sqrt{3}-i)}\cdot\frac{(\sqrt{3}+i)}{(\sqrt{3}+i)}$$

$$\frac{-2i\sqrt{3}+2}{3+1}$$

$$\frac{-2i\sqrt{3}+2}{4}=\boxed{\frac{1}{2}-\frac{i\sqrt{3}}{2}}$$

In Trig Form:

$$\frac{z_1}{z_2}=\frac{2}{2}\left(\cos(270-330)+i\sin(270-330)\right)$$

$$=\cos(-60^\circ)+i\sin(-60^\circ)$$

$$=\boxed{\frac{1}{2}+-\frac{\sqrt{3}}{2}i}$$

$r_2=\sqrt{(\sqrt{3})^2+(-1)^2}=2$

$\theta_2=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=330^\circ$

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HOMEWORK

...due Monday along with 6.5b

6.5a (p. 456): 1-35 (3's, 5's, 7's); 37-57 (5's, 7's, 9's); 61; 63; 71; 79

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$z = r(\cos\theta + i\sin\theta)$

$z^2 =$

Poll

Dec 5-3:50 PM

Steps for using Mr. Abraham's DeMoivre's Theorem:

Example 6 p. 452
Use DeMoivre's Theorem to find $(-1+\sqrt{3}i)^{12}$

p. 458, #91 (HW)

Dec 5-4:00 PM

Roots of Complex Numbers
Fundamental Theorem of Algebra:

$$x^4 = 1$$

Dec 5-4:05 PM

Can we find a formula that gives us ALL the roots?

nth Roots of a Complex Number
The nth roots of a complex number z=r(cosθ+isinθ) are given by:

where k=0, 1, 2,..., n-1

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Example p. 458, #126
(a) Find the square roots of the given complex number, (b) represent each of the roots graphically, and (c) write each of the roots in standard form.

$$16(\cos 60^o + i\sin 60^o)$$

p. 458, #125 (HW)

Dec 5-4:28 PM

Example 8 p. 455
Find the three cube roots of $z=-2+2i$

p. 458, #129 (HW)

Dec 5-4:31 PM

HOMEWORK

...due Monday along with 6.5a and 6.4-6.5 Partner Quiz

6.5b (p. 458): 91-105 (odd), 113, 115, 125-131 (odd), 141, 143

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