

Dec 3-8:13 AM

Angle Between Two Vectors \mathbf{u} and \mathbf{v} :

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} \quad \text{or} \quad \theta = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

Q: What would need to be true in order for \mathbf{u} and \mathbf{v} to be perpendicular?
 (Hint: What would θ need to equal?)

A: 90° angle between the vectors

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Look at the 5 different options for θ drawn on p. 440.
 We will focus on the 3rd possibility...

Definition. The vectors \mathbf{u} and \mathbf{v} are "orthogonal" (perpendicular) if

$$\vec{u} \cdot \vec{v} = 0$$

Example

Determine if the two vectors are orthogonal.

$$\vec{u} = \langle 10, 4 \rangle$$

$$\vec{v} = \langle 2, -5 \rangle$$

$$\vec{u} \cdot \vec{v} = (10)(2) + (4)(-5)$$

$$20 + -20$$

0
 Orthogonal

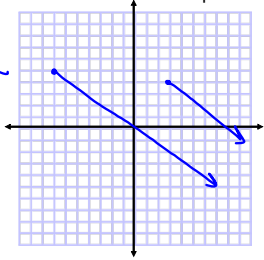
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Q: Can you figure out the condition necessary for two vectors \mathbf{u} and \mathbf{v} to be parallel? Hint: What do you notice about their slopes?

A: Slopes are same
Direction are same

$\theta = 180^\circ$ or 0°



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Example
Determine whether or not \mathbf{u} and \mathbf{v} are parallel.

$\vec{u} = \langle 8, 4 \rangle$
 $\vec{v} = \langle -2, -1 \rangle$

$\theta = \cos^{-1} \left(\frac{-20}{\sqrt{80} \cdot \sqrt{5}} \right)$

① $\vec{u} \cdot \vec{v} = 8(-2) + 4(-1) = -16 - 4 = -20$

② $\|\vec{u}\| = \sqrt{8^2 + 4^2} = \sqrt{64 + 16} = \sqrt{80}$

③ $\|\vec{v}\| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$

$\theta = \cos^{-1} \left(\frac{-20}{\sqrt{400}} \right) = \cos^{-1}(-1) = 180^\circ$

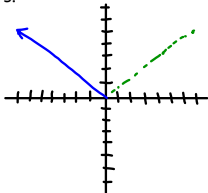
Parallel

p. 445: #33 (HW)

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Example
Find two vectors in opposite directions that are orthogonal to vector $\vec{u} = \langle -7, 5 \rangle$. There are many answers.

$\vec{v} = \langle 5, 7 \rangle$
 $\vec{v} = \langle -5, -7 \rangle$



p. 446, #53 (HW)

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HOMework
...due tomorrow; the underlined portion is what we completed today...

6.4 (p. 445): 1-31 (odd), 33-37 (odd), 53, 55

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