3.3: Properties of Logarithms

Review

(1) Convert to exponential form: $\log_b(M) = R$ $b^R = M$

(2) Use rules of exponents to simplify the following:

[A] $b^m \times b^n = b^{m+n}$ [B] $(b^m)^n = b^{mn}$

[C] $\sqrt[n]{b} = b^{\frac{1}{n}}$ [D] $\frac{b^n}{b^m} = b^{n-m}$

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Product Rule Recall $b^m b^n = b^{m+n}$ $\log_2 32$

$\log_2 8 + \log_2 4 =$ (3 + 2)	32	5	$= \log_2 32$
$\log_3 9 + \log_3 3 =$ (2, 1)	27	3	$= \log_3 27$
$\log_3 \frac{1}{9} + \log_3 81 =$ (−2 + 4)	9	2	$= \log_3 9$
$\log_5 5 + \log_5 1 =$ (1, 0)	5	1	$= \log_5 5$

Q: Can you generalize this idea?
A: Let b, M, and N be positive real numbers with $b \neq 1$.
Then $\log_b(M) + \log_b(N) = \log_b(MN)$

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Example 1
Use the Product Rule to expand; simplify if possible.

[A] $\log_6(7 \times 11) = \log_6 7 + \log_6 11$

[B] $\log(100x) = \log 100 + \log x$
$2 + \log x$

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Quotient Rule
Recall $\frac{b^m}{b^n} = b^{m-n}$

Let b, M, and N be positive real numbers with $b > 0, b \neq 1$ Then

$\log_b M - \log_b N = \log_b \frac{M}{N}$

Notice: The Product Rule: $\times$ on inside → $+$ on outside
The Quotient Rule: $\div$ on inside → $-$ on outside

Example 2 $\log_e e^5$
Use the Quotient Rule to expand:

[A] $\log_8\left(\frac{23}{x}\right) =$ [B] $\ln\left(\frac{e^5}{11}\right) =$

$\log_8 23 - \log_8 x$ $\ln e^5 - \ln 11$
$5 - \ln 11$

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Power Rule
Recall $(b^R)^P = b^{RP}$

Let b, M, and p be positive real numbers with $b \neq 1$. Then
$\log_b M^p = p\log_b M$.

Example 3
Use the Power Rule to expand:

[A] $\log_6 3^9 = 9\log_6 3$ [B] $\ln \sqrt[3]{x} =$
$\ln x^{\frac{1}{3}} = \frac{1}{3}\ln x$

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In symbols
$\log_b(M \times N) = \log_b M + \log_b N$
$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
$\log_b M^p = p\log_b M$

In words
The logarithm of a product is the sum of the logarithms.
The logarithm of a quotient is the difference of the logarithms.
The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number.

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Example 4
Expand as much as possible.

[A] $\log_b(x^4\sqrt[3]{y}) = \log_b x^4 + \log_b y^{\frac{1}{3}}$

$4\log_b x + \frac{1}{3}\log_b y$

[B] $\log_5\left(\frac{\sqrt{x}}{25y^3}\right) =$

$\log_5 x^{\frac{1}{2}} - \log_5 25 - \log_5 y^3$

$\frac{1}{2}\log_5 x - 2 - 3\log_5 y$

p. 211, #19, #37 (HW)

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Example 5
Condense into a single log.

[A] $\log 25 + \log 4 = \log 100 = 2$

[B] $\log(7x+6) - \log x = \log \frac{7x+6}{x}$

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[C] $2\ln x + \frac{1}{3}\ln(x+5) = \ln x^2 + \ln\sqrt[3]{x+5} = \ln\left(x^2\sqrt[3]{x+5}\right)$

[D] $2\log(x-3) - \log x = \log(x-3)^2 - \log x = \log\frac{(x-3)^2}{x}$

[E] $\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y =$

$\log_b \sqrt[4]{x} - \log_b 25 - \log_b y^{10}$

$\log_b\left(\frac{\sqrt[4]{x}}{25y^{10}}\right)$

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Example 7
Use the change-of-base property and your calculator to evaluate $\log_7 2506$.

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

$\log_7 2506 = \frac{\log 2506}{\log 7} = 4.022$

p. 211, #1 (HW)

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Wrap-Up
[A] Decide whether the following are true or false.

- $\boxed{\log\sqrt{2}} = \frac{\log 2}{2}$ $\quad \log 2^{\frac{1}{2}} = \frac{1}{2}\log 2 = \frac{\log 2}{2}$
  TRUE
- $\boxed{\log_a 14 + \log_a x} = \log_a 14 \times \log_a x$ $\quad$ False
  $\log_a 14x$
- $(\log_a M)^p = p\log_a M$
  TRUE

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HOMEWORK

~~3.1-3.2b and 3.2 worksheet due tomorrow~~

3.3 (p211):
1-25 (1,5,9's), 37-55 odd, 59-75 (eoo), 81-87 odd, 95

Project due Friday...Get R Dun

Feb 9-9:17 AM

$log_b(M) + log_b(N) =$ ____________________

Proof: (Don't be scared!)

Assume these two statements hold:

$$\boxed{\begin{array}{l} b^R = M \\ b^S = N \end{array}}$$

Rewrite both as log equations over here:

§ Multiply left sides and right sides to make one equation:

§ Combine those exponents:

§ Rewrite as a log equation:

§ Make a substitution and voila:

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$\log_b M^p = p\log_b M$.

Proof:

Assume this statement holds:

$$\boxed{b^R = M}$$

Rewrite as log equation over here:

- Raise both sides to the power of p:
- Combine those exponents:
- Rewrite as a log equation with base b:
- Make a substitution and voila:

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Change-of-Base Property

The Change-of-Base Property states:

For any log bases *a* and *b* and any *M* > 0,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

Proof:

Assume this statement holds:

$$\boxed{b^R = M}$$

Rewrite as log equation over here:

§ Take $\log_a$ of both sides:

§ Apply the Power Rule to the left side:

§ Solve for R:

§ Make a substitution and voila:

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[B] Given that:

$\log_b 3 \approx 1.0099$, and
$\log_b 5 \approx 1.609$,

Find $\log_b 15b$.

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